



Sesión Especial 5

Avances recientes en mecánica cuántica relativista

Organizadores

- Naiara Arrizabalaga (Universidad del País Vasco, UPV/EHU)
- Albert Mas (Universitat de Barcelona)
- Luis Vega (BCAM y Universidad del País Vasco, UPV/EHU)

Descripción

La ecuación de Dirac fue planteada a finales de los años 20 por el físico Paul Dirac y aúna dos de las ideas más importantes de la física: la mecánica cuántica, que describe el comportamiento de los objetos minúsculos, y la teoría de la relatividad, que describe el comportamiento de los objetos que se mueven a velocidades cercanas a la de la luz. El estudio de la ecuación de Dirac está en pleno auge debido a su relevancia física y a sus aplicaciones, entre las que se encuentra por ejemplo la predicción de la existencia de la antimateria o la modelización del comportamiento cuántico del grafeno. El propósito de esta sesión es juntar a jóvenes investigadores y a investigadores con una reconocida trayectoria, todos ellos expertos en la ecuación de Dirac y sus variantes. En la sesión se expondrán algunos de los avances más recientes en la materia y se propondrán problemas abiertos referentes al análisis y las propiedades espectrales de la ecuación.

Programa

LUNES, 4 de febrero (mañana)

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|---------------|--|
| 11:30 – 12:30 | Andrea Posilicano (Università degli studi dell'Insubria)
<i>Limiting absorption principle and scattering matrix for Dirac operators with shell interactions</i> |
| 12:30 – 13:30 | Hanne Van Den Bosch (CMM, Universidad de Chile)
<i>Boundary value problem for Dirac operators on corner domains</i> |

LUNES, 4 de febrero (tarde)

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| 17:00 – 18:00 | Vladimir Lotoreichik (Nuclear Physics Institute, Czech Academy of Sciences)
<i>The spectral gap of the 2-D Dirac operator with infinite mass boundary conditions</i> |
| 18:00 – 19:00 | Thomas Ourmières-Bonafos (CEREMADE, Université Paris-Dauphine)
<i>Dirac operators on surfaces as large mass limits of Euclidean Dirac operators</i> |



MARTES, 5 de febrero (mañana)

- 11:30 – 12:30 Alessandro Michelangeli (SISSA)
Recent problems on Dirac-Coulomb operators
- 12:30 – 13:30 Margherita Nolasco (Università degli Studi dell'Aquila)
Ground state for the relativistic one electron atom

MARTES, 5 de febrero (tarde)

- 17:00 – 18:00 Loïc LeTreust (Université d'Aix-Marseille)
On the semiclassical spectrum of the Dirichlet-Pauli operator
- 18:00 – 19:00 Anne-Sophie de Suzzoni (Université Paris XIII)
The Dirac equation on spherically symmetric manifolds

The Dirac equation on spherically symmetric manifolds

ANNE-SOPHIE DE SUZZONI

Université Paris XIII

adesuzzo@math.univ-paris13.fr

Abstract. In this talk, we will present the Dirac operator on smooth spherically symmetric manifolds and explain how it can be diagonalized onto radial matrix operators. We then use this diagonalization and Littlewood-Paley theory on the sphere to introduce local Strichartz estimates for the linear Dirac equation. We then present an application to non-linear Dirac equations, valid in particular on hyperbolic manifolds. We explain for instance why the Dirac equation is locally well-posed if the initial datum is radial and taken in H^s for $s > 1$.



On the semiclassical spectrum of the Dirichlet-Pauli operator

LOÏC LETREUST

Université d'Aix-Marseille

loic.le-treust@univ-amu.fr

Abstract. This talk is devoted to semiclassical estimates of the eigenvalues of the Pauli operator on a bounded open set whose boundary carries Dirichlet conditions. Assuming that the magnetic field is positive and a few generic conditions, we establish the simplicity of the eigenvalues and provide accurate asymptotic estimates involving Bergman-Hardy spaces associated with the magnetic field.

The spectral gap of the 2-D Dirac operator with infinite mass boundary conditions

VLADIMIR LOTOREICHIK

Nuclear Physics Institute, Czech Academy of Sciences

lotoreichik@ujf.cas.cz

Abstract. We will discuss the massless Dirac operator D_Ω on a bounded, convex, C^3 -domain $\Omega \subset \mathbb{R}^2$ with so-called infinite mass boundary conditions. This Dirac operator arises in an effective mathematical theory for graphene. The operator D_Ω is self-adjoint in $L^2(\Omega, \mathbb{C}^2)$ and non-semi-bounded. Its spectrum $\sigma(D_\Omega)$ is discrete and symmetric with respect to the origin. The size of the spectral gap for D_Ω

$$\mathcal{L}_\Omega := \text{dist}(0, \sigma(D_\Omega)) > 0$$

is known to be important in applications. Our main result concerns the geometric control on \mathcal{L}_Ω . Namely, we obtain an upper-bound on \mathcal{L}_Ω in terms of $\mathcal{L}_\mathbb{D}$ for the unit disk \mathbb{D} and of an explicit geometrically-induced pre-factor. This result can also be reformulated as a reversed Faber-Krahn-like inequality for D_Ω under a proper geometric constraint.

This talk is based on a joint work with Thomas Ourmières-Bonafos.



Recent problems on Dirac-Coulomb operators

ALESSANDRO MICHELANGELI

SISSA

alessandro.michelangeli@sissa.it

Abstract. I will survey recent results and open problems on one-body and many-body Dirac-Coulomb systems, concerning their non-ambiguous realisation as self-adjoint Hamiltonians, their possible perturbations of point-like nature, their stability, and other spectral features.

Ground state for the relativistic one electron atom

MARGHERITA NOLASCO

Università degli Studi dell'Aquila

nolasco@univaq.it

Abstract. We study the Dirac-Maxwell system coupled with an external potential of Coulomb type. We use the Foldy-Wouthuysen (unitary) transformation of the Dirac operator and its realization as an elliptic problem in the 4-dim half space with nonlinear Neumann boundary condition. Using this approach we study the existence of a “ground state” solution.

Dirac operators on surfaces as large mass limits of Euclidean Dirac operators

THOMAS OURMIÈRES-BONAFOS

CEREMADE, Université Paris-Dauphine

thomas.ourmieres-bonafos@u-psud.fr

Abstract. In this talk, I will discuss a new link we recently made between Dirac operators in bounded Euclidean domains and Dirac operators on manifolds. Namely, we will see that the eigenvalues of the so-called intrinsic Dirac operator on the boundary of a Euclidean domain can be obtained as the limits of Euclidean Dirac operators, either in the domain with a MIT-bag type boundary condition or in the whole space, with a suitable chosen zero order mass term.

This is joint work with A. Moroianu and K. Pankrashkin.



Limiting absorption principle and scattering matrix for Dirac operators with shell interactions

ANDREA POSILICANO

Università degli studi dell'Insubria

Andrea.Posilicano@uninsubria.it

Abstract. We provide a limiting absorption principle for self-adjoint realizations of Dirac operators with shell interactions supported on compact surfaces. Then we show asymptotic completeness of the wave operators and give a representation formula for the scattering matrix.

Joint work with Jussi Behrndt, Markus Holzmann and Andrea Mantile.

Boundary value problem for Dirac operators on corner domains

HANNE VAN DEN BOSCH

CMM, Universidad de Chile

hannevdbosch@gmail.com

Abstract. The goal of this talk is to present results on the self-adjoint extensions of Dirac operators on plane domains with corners. We consider the case of general boundary conditions for a quantum dot (confined particle), and a special type of delta-shell interaction. For these cases we obtain explicitly the self-adjoint extensions of the operator. It turns out that the presence of corners typically spoils the elliptic regularity known to hold for smooth boundaries.

This is joint work with Fabio Pizzichillo and Luis Vega.